An improvement of the upper radiative boundary condition

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1. Introduction
Since mesoscale numerical models resolve vertically propagating gravity waves, they need an upper boundary condition that prevents spurious reflection of these waves. Currently, two different methods are available. The first one is to use a thick damping layer in the upper part of the model domain that gradually dissipates the wave energy, often called a sponge layer. The damping may either be accomplished with enhanced horizontal diffusion (e.g., Klemp and Lilly 1978) or with linear relaxation to a given basic state (e.g., Saito and Iwasa 1991). Alternatively, as is done in the MM5, a radiative upper boundary condition may be used. The main advantage of a radiation condition is its efficiency. While a sponge layer must extend over more than 10 model levels to yield satisfactory results, only one level is needed for a radiation condition.

Each of these methods has its limitations. A diffusive sponge is useful only if the horizontal resolution is not finer than ~10 km. At higher resolutions, a significant part of the gravity wave spectrum will not be damped properly. A relaxation to a basic state requires knowledge of this basic state. This is straightforward for idealized simulations, but in general, the basic state has to be computed by some horizontal and vertical averaging or to be interpolated from the NWP analysis data with which the mesoscale model is driven. The most severe weakness of the radiative boundary condition is—similar to the diffusive sponge—the limited range of horizontal wavelengths captured by it (see below). In this paper, a way to overcome this limitation with little additional computational costs is presented.

2. The problem
Since the upper radiative boundary condition (i.e., the vertical wind field in the uppermost model level) has to be computed non-locally by means of a Fourier transform, a local subdomain around each grid point is needed. The extension if this subdomain determines the range of horizontal wavelengths captured by the radiation condition. In the MM5 model, a 13×13 grid centered at each grid point is used. Consequently, the maximum horizontal wavelength that is able to radiate upward is 12 times the grid distance when the wavenumber vector is aligned with the grid. In general, the maximum wavelength is larger by a factor of \( \frac{\cos \alpha}{\Delta \varphi} \) where \( \alpha \) is the smallest angle between the wavenumber vector and one of the coordinate axes. This yields a theoretical maximum of \( 12\sqrt{2}\Delta x(\approx 17\Delta x) \). Longer waves are reflected back in the same way as if a rigid lid was present at the model's top.

It is easy to see that this wavelength range is insufficient for high-resolution simulations. For typical atmospheric parameters, vertical gravity wave propagation is possible up to wavelengths of several hundreds of kilometers, Coriolis effects being of minor importance up to ~150 km. Thus, at least for grid distances below 10 km, a significant part of the gravity wave spectrum is reflected back. For illustration, a simulation of idealized flow over a quasi-two-dimensional mountain is presented in Fig. 1. The mountain shape is defined by

\[
h(x) = \frac{h_o}{(1 + \frac{a}{\Delta x})^{1.5}},
\]

where \( h_o \) and \( a \) being 300 m and 40 km, respectively. A uniform flow with \( U = 10 \, \text{m s}^{-1} \) and \( N = 10^{-2} \, \text{s}^{-1} \) is prescribed, and Coriolis effects are turned off. Evidently, the model fails to reproduce the characteristic upstream phase shift of vertically propagating gravity waves.

3. An efficient solution
The modification presented here makes use of the
For horizontal resolution \( \Delta x \) \( \lesssim 1 \) km since gravity waves for horizontal resolutions \( \Delta x \) \( \gtrsim 1 \) km cannot propagate on a domain with grid spacing \( \Delta x \) \( \lesssim 1 \) km. This option is applicable because a vertical resolution \( \Delta z \) \( \gtrsim 1 \) km is required for the resolution condition for waves to be propagated horizontally. Therefore, a reduced resolution condition can be employed for higher resolutions. In this case, the reduced resolution condition for waves is imposed to model the high-resolution radiation condition. A more refined estimate is obtained with the following condition:

\[
(\Delta x \Delta z)^2 \lesssim (\Delta x)^3
\]

The result obtained with the reduced resolution condition is displayed in Fig. 2. Obviously, a substantial improvement is achieved. The result obtained with the reduced resolution condition can be calculated by the following equation:

\[
(\Delta x \Delta z)^2 \lesssim (\Delta x)^3
\]

Thus, the radiation condition is implemented as a 2D Fourier transform. To obtain the desired resolution, the radiation condition must be applied iteratively. The grid spacing increases by a factor of 3 for each iteration. In this case, the radiation condition is interpolated from the next coarser domain without filtering. This option is appropriate for horizontal resolutions

\[
\lesssim 30 \text{ km}
\]
shorter than \( \sim 5.10 \text{ km} \) (depending on atmospheric conditions) are vertically damped and thus are not treated properly with a radiation condition.

4. Suggestions for further improvements

The smoother-desmoother applied to the meteorological fields after feedback from a nest to a coarser domain causes appreciable smoothing of wavelengths between \( 4\Delta x \) and \( 6\Delta x \). Thus, the amplitude of the interpolated radiation condition is too low in the corresponding wavelength range. This may cause partial wave reflections. Probably, a further improvement of the radiation condition can be attained by enlarging the subdomain used for its computation to \( 19 \times 19 \) grid points. Then, only wavelengths \( \geq 6\Delta x \) have to be interpolated from the next coarser domain. However, a disadvantage of enlarging the subdomain is that the “bands” near each lateral boundary where computing the radiation condition is not possible enlarge, too. Thus, the current \( 13 \times 13 \) grid should be retained near the lateral boundaries.

In addition, Coriolis and nonhydrostatic effects should be taken into account. While the presence of Coriolis force reduces the vertical wavelength of gravity waves, nonhydrostatic effects increase it substantially. This has severe implications for the phase shift between the two uppermost model levels that is computed by the radiation condition. In particular, there is no phase shift at all for vertically damped gravity waves, i.e., for horizontal wavelengths less than \( 2\pi \frac{U}{N} \).

References
